Chapter

Optimized Portfolios: All Seasons Strategy

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Abstract

Our study explores the efficient frontier of optimal investment, taking behind the Markowitz's theory, while advocating a diversified portfolio to reduce risk. To perform it, six portfolio models are proposed, and its formation are made by a solver, where the selected solving method is the GRG Nonlinear engine for linear solver problems. Our main goal is to design portfolios that resists to financial crisis but at the same time persists in a wealthy period. We analyze the decade where we assisted to two crashes (2000–2010) and a semi-decade where we assist to a wealthy period (2011–2018). The assets used are varied, such as Equities indexes form various countries, sector equities, bonds, commodities, EURUSD exchange and VIX. Results show that the GRG Nonlinear engine is powerful, providing excess returns in all six models.

Keywords: MPT, Markowitz, portfolios' formation, sharp-ratio, volatility

1. Introduction

The inspiration for our work comes from the well-known investor, Ray Dalio who built a considerable personal fortune with the incredible success of the Pure Alpha strategy. In the mid-1990s he began to think about his inheritance and funds he wanted to leave behind and asked this question: "What kind of portfolio would you use if you were not already present to actively manage money?" What kind of portfolio would survive your own decision-making and would continue to support their children and their philanthropic efforts for decades? [1].

A brand-new look at asset placement. A new set of rules. And only after the portfolio has been retrospectively tested until 1925; only after having produced consistent results in a variety of economic conditions, Ray Dalio began to offer it to a narrow group. The new strategy, known as the "All Seasons" strategy, was publicly unveiled in 1996, just four years before a mass market correction put it to the test. "Passed" with distinction [1].

Conventional wisdom and the conventional management of a portfolio leave us in the hands of a model that continually shows that it cannot survive when times are tough. So, we began to explore whether we could define portfolios - asset distribution - that would perform well in any economic environment in the future, such as in the year 2008, a depression or a recession. Because no one knows what is going to happen in five years, how much more in 20 or 30 years.

According to [1], having into account this basis, we propose six different models, aiming to maximize returns but at the same time, reduce risk. Theory behind this is the Harry Markowitz's [2, 3], who is known as the father of modern portfolio theory. It explains in this way and synthesized the fundamental concept behind the work that earned him the Nobel Prize: investments in a portfolio should not be seen individually, but as a group. There is a trade-off between risk and return, so "do not listen to just one instrument, listen to the entire orchestra". How investments behave together and how they are diversified will determine return. This advice may seem simple now, but in 1952 this thought was groundbreaking. Somehow this approach influenced virtually all portfolio managers from New York and Hong Kong.

We combine portfolios with a wide range of equity (mainly indexes from various countries and the main sectors as well), different kind of bonds (US and German treasury bonds and corporate bonds), a range of commodities (for example, different metals, agriculture commodities, energy commodities, etc.), EUR/USD exchange and VIX (volatility index of S&P 500) through a solver using the GRG (Generalized Reduced Gradient) Nonlinear engine for linear solver problems. The spam range is from 2000 to 2018 in order to cover two market crashes (2002 – technological and 2008 – subprime) and a good decade forward. Our mains investigation question is if it is possible to create a portfolio or a set of portfolios that presents robust results in a bad decade but, at the same time, in a good decade as well? Results show that definitively, is doable.

Next section, literature review, we explore the theory behind the concept of this study and empirical achievements from different authors. Section 3 presents six different models where we are going to using the solver, Section 4 preliminary analysis to the data set, in Section 5 we present the results of the models and we propose some portfolios to use as well. Finally, Section 6 concludes.

2. Literature review

Many investors are naive in their financial beliefs and do not understand basic concepts such as equity or diversification [4, 5]. Benjamin Graham 1949 *apud* [6], the father of the value investing, proposed that an equilibrated portfolio should be constituted by 50% equity and 50% bonds; an intelligent investor may own 100% equity in his portfolio in certain conditions, the most important of them: only if in a crash crisis, the portfolio presented a positive return. By dividing the money by 50% for stocks and 50% for bonds (or some similar variation), many investors would think they were diversifying and reducing their risk. But later, when [2] presented his work about the efficient portfolio, concluded that what investors are doing is taking more risks than they think. Because, according to Ray Dalio *apud* [1–3] shares can be three times riskier (i.e., volatile) than bonds. In fact, by having a 50/50 portfolio, we have something more like a 95% risk distribution in stocks. Below, **Figure 1** represents a chart with a 50/50 portfolio. The left side shows the money divided by shares and bonds, in percentage. The right side shows the same portfolio, but divided in terms of risk, between stocks and bonds.

At first glance, with 50% of the money in shares, it seems relatively balanced. But, as it turns out here it would have been about 95% risk, given the volatility of its composition in stocks. So, if shares sink, the whole portfolio sinks. And the balance is lost. How does this concept work into real life? From 1973 to 2013, the S&P 500 lost money nine times and accumulated losses totaled 134%. In the same period, bonds (represented by the Barclays Aggregate Bond index) lost money only three times and accumulated losses were 6%. Therefore, having a 50/50 portfolio, the S&P 500 would have accrued 95% of the losses.

Placing assets is the only key that can differentiate us from all investors [2, 3]. Nobel Prize winner and father of modern portfolio theory (MPT) said that "diversification is the only free lunch." Why? Because spreading the money for different investments lowers the risk and increases the possibility of gains over time and costs nothing.

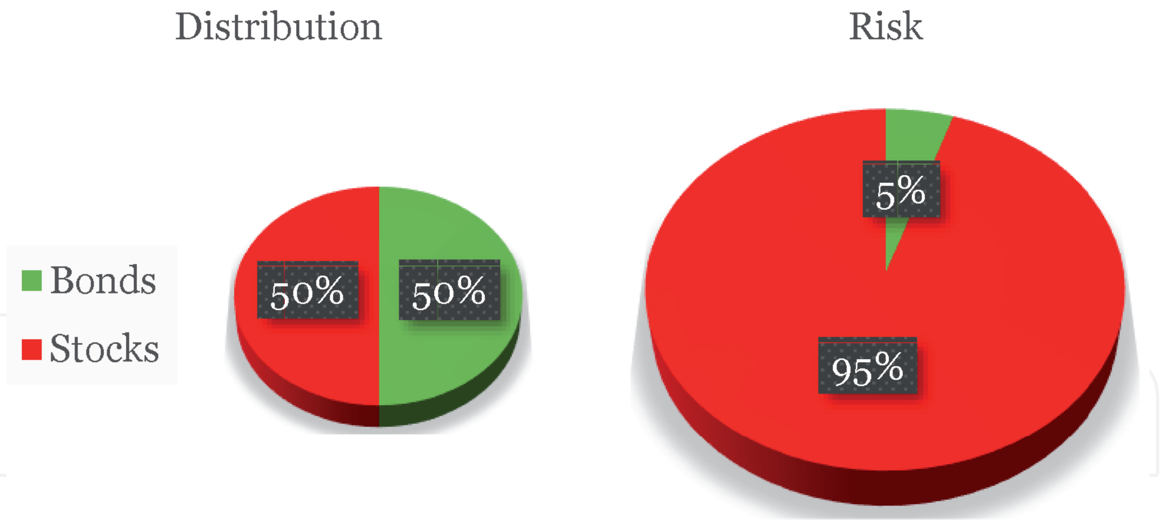

Figure 1.
Allocation versus risk. Source: adapted from [1].

	GROWTH	INFLATION
RISE ↑	Higher than expected economic growth	Higher than expected inflation
FALL ↓	Lower than expected economic growth	Lower than expected inflation

Table 1.
The four things that drive asset prices. Source: adapted from [1].

When we look at most portfolios, they usually hold up well in good periods, but they fall in bad periods. And then, the strategy is simply to wait for the stock to go up. This conventional approach to diversifying investments is not at all diversified (Dalio *apud* [1]). According to [7] "Financial crises occur in all market economies, although sometimes there are long periods of quiet. Crises occur in developed countries, not just emerging markets. Crises occur in economies with and without a central bank and with and without deposit insurance".

Competitive pressures and market efficiency turn difficult to financial forecast - particularly to predict asset returns - is very difficult compared to standard forecasting problems in macroeconomics, in which the presence of a sizeable persistent component makes forecasting easier [8].

Dalio *apud* [1] revealed the simplest and most important distinction of all. There are only four things that drive asset prices (**Table 1**):

1. Inflation

2. Deflation

3. High economic growth

4. Declining economic growth

In this way, it all boils down to four possible environments, or economic "seasons," which will ultimately determine whether investments (asset prices) rise or fall - except that, unlike the seasons, there is no predetermined order in the succession. They are:

1. Inflation higher than expected (rising prices)

2. Inflation lower than expected (deflation)

3. Economic growth higher than expected

4. Economic growth lower than expected

The price of a stock (or a bond) already incorporates what we (the market) "expect" about the future. Many authors [9–11], claim that there is literally a picture of the future when looking at prices today. In other words, the stock price of a company today already incorporates the expectations of investors, who believe that the company will continue to grow at a certain pace [12–14] – this phenomenon also known as efficient market hypothesis (EMH). This is why is sometimes heard that the stock price will fall when companies announce that their future growth (their profits) will be lower than they had originally forecast – see also the post-earnings announcement drift (PEAD) phenomenon [15–17].

It is the surprises that will ultimately determine which asset class will behave well. If the news announces that there will be sustained growth, this will be very good for stocks and not so good for bonds. If we watch a surprise fall in inflation this will be good for the bonds [18, 19]. If there are only four potential economic environments, or seasons, one should therefore have 25% of the risk in each of the categories. That is why this approach is called "All Seasons" because there are four possible seasons in the financial world, and no one really knows which season will come next – EMH/Random walk [12, 20–22]. With this approach each season, each quadrant is always covered, so the portfolio is always protected. Let us imagine, then, four portfolios, each with an equivalent amount of risk. This means that we will not have exposure to any particular environment. We are not trying to predict the future, because no one knows what the future will bring [12, 22–24]. What is known is that there are only four potential seasons that we will have to face. Using this investment strategy, we can know that we are protected - not just hopeful - and that the investments are safe and will perform well in any season that comes.

"All Seasons": today we can structure a portfolio that will behave well in 2029, even if we have no chance of knowing what the world will look like in 2029. Below is a table that shows the four potential seasons and the type of investment that will perform best in each of these environments, categorizing each of them in each of the seasons (**Table 2**).

The original "All Seasons" is composed by equity, bonds and commodities which became a popular asset over the past decade [25]. [26] argues that MPT is the formula of diversification, which selects a collection of assets that has collectively lower risk than individually. In sum, for a given amount of risk, MPT describes how to select a portfolio with the highest possible expected return [27, 28]. Below it is presented, in **Figure 2**, the efficient frontier.

	GROWTH	INFLATION
RISE ↑	Equities Corporate bonds Commodities/gold	Commodities/gold TIPS
FALL ↓	Treasury bonds Treasury Inflation-Protected Securities (TIPS)	Treasury bonds Equities

Table 2.
List of assets for each "season".

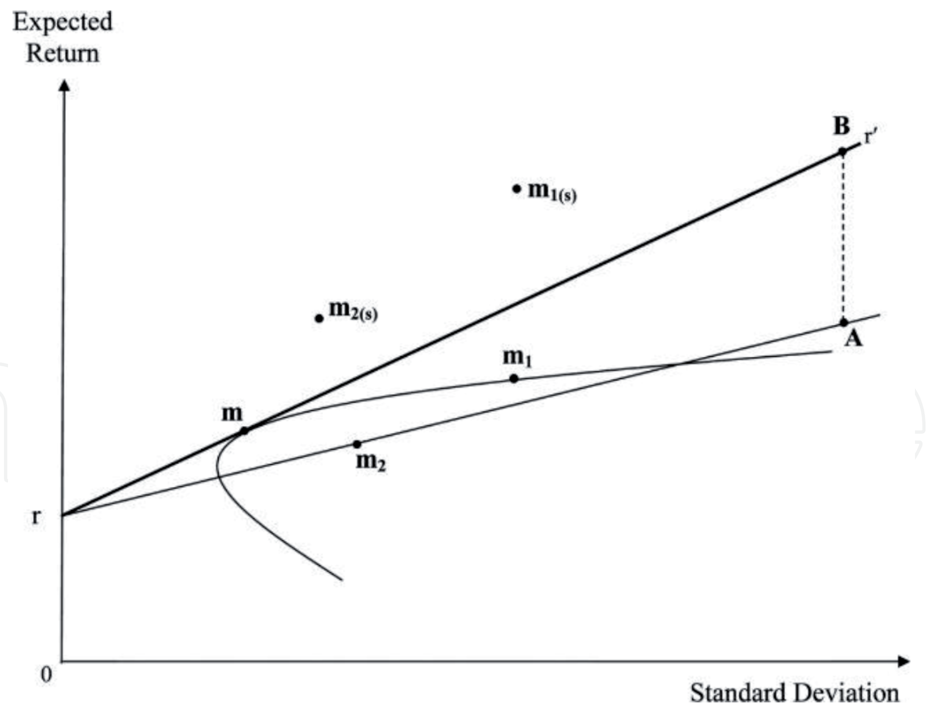

Figure 2.
Efficient frontier. Source: [29].

The hyperbola is sometimes referred to as the "Markowitz Bullet" and is the efficient frontier if no risk-free asset is available. With a risk-free asset, the straight line is the efficient frontier.

The Capital Asset Pricing Model (CAPM), for example, was the next step, it approached the risk of an individual asset through the diversification theory [30].

Based on this theory background and MPT, we present in the next chapter six different portfolios aiming to a certain risk, produce the maximum return to the investor.

3. Model framework

Six portfolio models are proposed: first, it is used a solver, where the selected solving method is the Generalized Reduced Gradient (GRG) Nonlinear engine for linear solver problems. The form is:

$$\max f(x): h(x) = 0, L \le x \le U, \tag{1}$$

Where h has dimension m. The method supposes can be partition $x = (v,w)$ such that:

- v has dimension m (and w has dimension $n\text{-}m$);
- the values of v are strictly within their bounds: $L_v < v < U_v$ (this is a nondegeneracy assumption);
- $\nabla_v h(x)$ is nonsingular at $x = (v,w)$.

As in the linear case, for any w there is a unique value, $v(w)$, such that $h(v(w),w) = 0$ (c.f., Implicit Function Theorem), which implies that

Equities index	Bonds	Commodities	Other
S&P 500	Treasury 1–3 years	Cocoa	EUR/USD
Dow Jones 30	Treasury 7–10 years	Coffee	VIX
NASDAQ	Treasury 20+ years	Corn	
EuroStoxx 600	TIPS	Sugar	
Hang Seng	Corporate bonds	Gold	
Emerging Markets	Bunds (Germany)	Copper	
Real Estate		Silver	
Consumer		Crude	
Healthcare		Natural Gas	
Communications		General commodities index	
Energy			
Financials			
Industrials			
Semiconductors			

Notes: S&P = Standard & Poor's; TIPS = Treasury Inflation Protected Securities; EUR/USD = Euro vs. USA Dollars exchange; VIX = Volatility Index (S&P 500).

Table 3.
Assets list.

$dv / dw = \left(\nabla_v h(x)\right)^{-1} \nabla_w h(x)$. The idea is to choose the direction of the independent variables to be the reduced gradient: $\nabla_w \left(f(x) - y^T h(x)\right)$, where $y = dv / dw = \left(\nabla_v h(x)\right)^{-1} \nabla_w h(x)$. Then, the step size is chosen, and a correction procedure applied to return to the surface, $h(x) = 0$.

The main steps (except the correction procedure) are the same as the reduced gradient method, changing the working set as appropriate.

The constitution of the portfolios is set from the Solver and varies. The six portfolios present different risks and returns, depending of the profile of each investor. There are conservative portfolio and aggressive portfolios. The solver configuration of each portfolio is showed above.

With regards to variable cells, the percentage of weighing of the assets type are the changeable ones. It is used a wide asset as equity indexes, bonds, commodities and other. The detail of each family of assets used in the model are listed table above. In total, it is used 32 assets:

With regards to subject to the constraints, the sum of the percentage of each asset is equal to 1, i.e., 100%:

$$\sum x \; Assets(a) = 1 \tag{2}$$

Where x = coefficient; a = each type of asset as showed in **Table 3** – Assets list.

Note: it is forced to make unconstrained variables non-negative.

The period is set between 2000 to 2018, but in some analysis the two decades are separated (2000 to 2010; and 2011–2018). The reason of the period spam used is important because:

1. The first decade (2000 to 2010) was very turbulent for financial markets, where occurred two crashes:

    a. Between 2001 and 2002 the technological crash;

    b. Between 2008 and 2009 the subprime crisis.

2. The second semi-decade not completed yet, between 2011 to 2018 where there is a recover from the last decade.

Then is important to study some robust portfolios that can provide some return to investors and at the same time with lower risk, mean, volatility, in order to be prepared for crashes or deflationary periods.

The model uses past returns (monthly returns) for each asset and the portfolios are re-equilibrated monthly according to the optimal weighting of each one. The benchmark, to compare results is the S&P 500 index. It is relied on monthly returns, computed as given by:

$$R_t = \frac{P_t}{P_{t-1}} - 1 \tag{3}$$

Where R_t = monthly returns; P_t and P_{t-1} are the assets prices at moments *t* and *t-1* respectively.

Finally, it is presented above, for each model, the own specifications and objectives of each one:

3.1 Model 1: maximize sharp ratio

The set objective of this model is to maximize the sharp ratio for all the period (2000–2018). It is relied by the division by the average year return and the standard deviation, computed as given by the following steps:

$$S = P(1+i)^n \tag{4}$$

Where S = Accumulated value; P = Principal.

That is:

$$FV = PV(1+i)^n \tag{5}$$

Where FV = Future Value; PV = Present Value; i = rate; n = number of periods (years).

But is need the rate for the numerator:

$$i = \sqrt[n]{\frac{FV}{PV}} \tag{6}$$

The, the standard deviation (σ) – the denominator – is a measure of how widely values are dispersed from the average value (the mean), using the "n" method. It is used the following formula:

$$\sigma = \sqrt{\frac{\Sigma\left(x - \bar{x}\right)^2}{n}} \tag{7}$$

Finally, the Sharp Ratio (SR) formula:

$$SR = \frac{i}{\sigma} \tag{8}$$

This is a measure of stability, if SR > 1 it means that returns overcome the standard deviation (volatility).

3.2 Model 2: maximize rate return

In this model, the concern is to bring the maximum return to the investor, ignoring the volatility, then we can argue that model 2 presents the higher risk comparing to others:

Set objective: Global rate return
To: Maximum

3.3 Model 3: two decades, equals returns

The model equals the return of the two decades ($i_{2000-2010} = i_{2011-2018}$). Comparing to other models, it may generate a more distributed income to investors. Then, in a decade of crises the investor may generate the same return as in a period of expansion. Also, model 3 "guarantees" a minimum return of half of percent each year:

Set objective: Global rate return
To: Maximum
Additional subject to the constrains: $i_{2000-2010} = i_{2011-2018}$; i_n > 0,5%

3.4 Model 4: maximize rate and sharp-ratio

In this model, the concern is to bring some extra return to the investor. It may generate more income than the model 1 but still with the concern of a stability, lowering a little bit the volatility of the portfolio:

Set objective: Global rate return
To: Maximum
Additional subject to the constrains: Sharp Ratio > 1 (all model)

3.5 Model 5: maximize rate and sharp-ratio (version 2)

In this model, in a similar way of the previous model (model 4), the concern is to bring some extra return to the investor but still with the concern of a stability, lowering a little bit the volatility of the portfolio:

Set objective: Global rate return
To: Maximum
Additional subject to the constrains: SR > 1 (period 2000–2010); SR > 1 (period 2011–2018)

Comparing to the previous model, the SR >1 appears twice in the constrains and not in the whole model (2000–2018). This measure will provide stability in the first decade but in the second decade as well.

3.6 Model 6: maximize the minimal year return

The model "guarantees" a minimal return year by year. Then, it may generate positive returns each year. Basically, it maximizes the minimum:

Set objective: Minimum return of each year (2000–2018)
To: Maximum
Additional subject to the constrains: Year return > Minimum return of each year

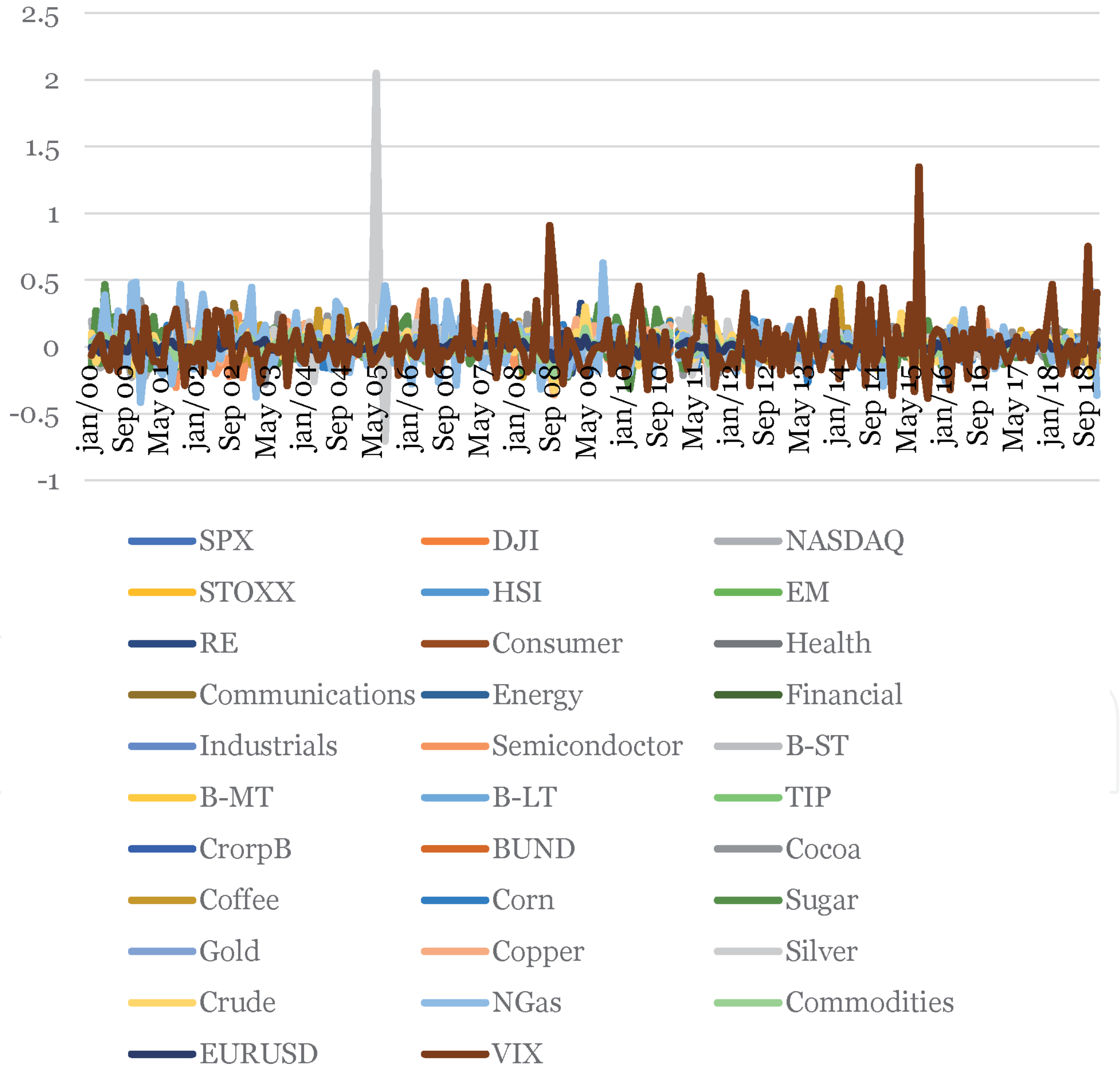

Figure 3.
Monthly returns of the 32 assets. Notes: SPX = Standard & Poor's ticker; DJI = Dow Jones Industrials ticker; STOXX = Eurostoxx 600 ticker; HSI = Hang Seng Index; EM = Emerging Markets; RE = Real Estate; B-ST = US Bonds Short Term; B-MT = US Bonds Medium Term; B-LT = US Bonds Long Term; TIP = Treasury Inflation Protected Securities; CorpB = Corporation Bonds; NGas = Natural Gas; EUR/USD = Euro vs. USA Dollars exchange; VIX = Volatility Index (S&P 500).

4. Preliminary data analysis

Market-adjusted prices data were collected from Yahoo Finance and from the Investing.com databases for all assets between 2000 and 2018. Monthly data for the assets informs the computation of returns. **Figure 3** reports the fluctuations of the months returns, illustrating the synchronized behavior of the returns compared with prices (**Figure 4**). Correlation matrix and collinearity statistic were made (table to big, then only available by request) and descriptive statistics of monthly returns of the assets in **Table 4**.

The clusters are quite evident: volatility is present during the period. It is noticed also that spikes vary in time and between the assets themselves which is expected according to the propose in this study in order to create an adequate and a stable portfolio for "four stations". In general, spikes are more evident in VIX, which means this is the asset with more variation in prices (volatility). We also can see two evident clusters in this asset during the crisis of 2008 and before October of 2015 (fears about China). It is noticed also that silver had an evident cluster after April of

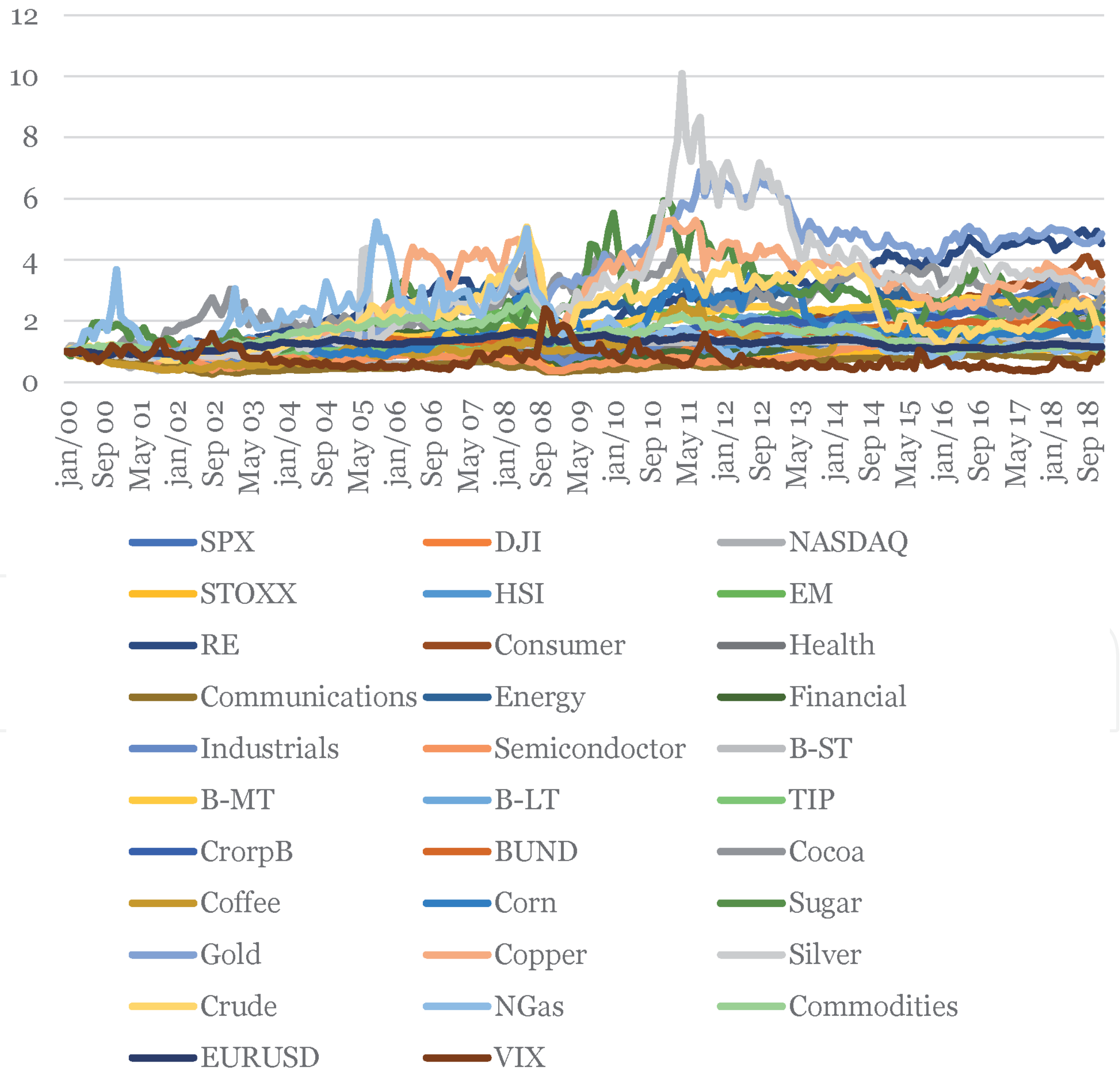

Figure 4.
Accumulated returns of the 32 assets. Notes: SPX = Standard & Poor's ticker; DJI = Dow Jones Industrials ticker; STOXX = Eurostoxx 600 ticker; HSI = Hang Seng Index; EM = Emerging Markets; RE = Real Estate; B-ST = US Bonds Short Term; B-MT = US Bonds Medium Term; B-LT = US Bonds Long Term; TIP = Treasury Inflation Protected Securities; CorpB = Corporation Bonds; NGas = Natural Gas; EUR/USD = Euro vs. USA Dollars exchange; VIX = Volatility Index (S&P 500).

	Min	Max	Mean	Standard Deviation	Var	Kurtosis
SPX	−0,16,942	0,107,723	0,003389	0,041935	,002	1208
DJI	−0,14,060	0,106,046	0,004066	0,040566	,002	1026
NASDAQ	−0,22,901	0,191,947	0,004314	0,063888	,004	1478
STOXX	−0,14,134	0,134,717	0,000475	0,043185	,002	,981
HSI	−0,19,218	0,191,929	0,003585	0,060497	,004	,506
EM	−0,25,578	0,168,629	0,005717	0,064815	,004	,757
RE	−0,30,435	0,325,359	0,008478	0,058845	,003	7569
Consumer	−0,16,678	0,135,928	0,006671	0,046783	,002	1075
Health	−0,14,249	0,085279	0,005354	0,035844	,001	1247
Communications	−0,19,721	0,323,605	0,000458	0,059830	,004	4326
Energy	−0,17,820	0,169,379	0,005666	0,055412	,003	,865
Financial	−0,22,757	0,217,848	0,004584	0,054803	,003	3504
Industrials	−0,19,954	0,194,630	0,005873	0,052299	,003	1912
Semicondoctor	−0,30,345	0,238,355	0,006824	0,079641	,006	1485
B-ST	−0,01401	0,023438	0,001383	0,003911	,000	6893
B-MT	−0,05473	0,077308	0,004492	0,017663	,000	1557
B-LT	−0,13,070	0,138,855	0,004450	0,035025	,001	3182
TIP	−0,08111	0,065035	0,002477	0,015140	,000	6606
CrorpB	−0,10,723	0,133,314	0,003885	0,019498	,000	12,713
BUND	−0,03512	0,047787	0,003007	0,015447	,000	-,203
Cocoa	−0,28,082	0,345,646	0,008779	0,093491	,009	,874
Coffee	−0,22,600	0,436,102	0,003457	0,090881	,008	2374
Corn	−0,26,536	0,221,904	0,005798	0,085273	,007	,433
Sugar	−0,31,247	0,463,178	0,008467	0,103,947	,011	1641
Gold	−0,18,005	0,138,671	0,008176	0,048522	,002	,752
Copper	−0,36,149	0,340,836	0,007984	0,076578	,006	3505
Silver	−0,70,670	2,047,420	0,015425	0,168,140	,028	96,160
Crude	−0,32,621	0,297,143	0,006479	0,092205	,009	,572
NGas	−0,41,616	0,626,133	0,012380	0,157,891	,025	1771
Commodities	−0,22,325	0,137,865	0,001168	0,047982	,002	1895
EURUSD	−0,09720	0,101,047	0,001066	0,029126	,001	1183
VIX	−0,38,489	1,345,709	0,019859	0,217,513	,047	6695

Notes: Min = Minimum; Max = Maximum; Var = Variance; SPX = Standard & Poor's ticker; DJI = Dow Jones Industrials ticker; STOXX = Eurostoxx 600 ticker; HSI = Hang Seng Index; EM = Emerging Markets; RE = Real Estate; B-ST = US Bonds Short Term; B-MT = US Bonds Medium Term; B-LT = US Bonds Long Term; TIP = Treasury Inflation Protected Securities; CorpB = Corporation Bonds; NGas = Natural Gas; EUR/USD = Euro vs. USA Dollars exchange; VIX = Volatility Index (S&P 500).

Table 4.
Descriptive statistics of monthly returns of the 32 assets.

2005 when other assets remained stable. What regards to equity, the most positive cluster (i.e. low spike) is present after May of 2009 when the market was recovering from the crisis.

If compared to the next figure (**Figure 4**), it is illustrated the synchronized behavior of the returns compared with prices. The spikes are much more evident. It also offers a clear picture of the volatility clusters.

What refers to correlation matrix and collinearity statistic (table available by request), US markets are very correlated and collineated with European market and all equity-sectors, although there is no correlation with the Chinese market, bonds, commodities and EUR/USD exchange. Also, it is shown that VIX is inverse correlated to the equity market in general. What regards to bonds, there is a high correlation and collinearity between themselves (except for the short-term bonds which are only a little correlated to medium-term bonds) but there is not (in general) with commodities and VIX. Commodities, in general, are not correlated with themselves (except crude oil that is correlated to all commodities index) and not correlated either to VIX or EUR/USD exchange. It is interesting to note that agriculture commodities are not correlated at all to themselves (cocoa, coffee, corn or sugar) but neither metals, for example, are not correlated between themselves (gold, copper or silver).

As the table above shows, standard deviation presents higher values rather the mean which means that volatility is present for all types of assets. Also, kurtosis presents value higher than 3 for Real State (equity), communications, financial services, short-term bonds, long-term bonds, TIPs, corporate bonds, copper, VIX and an exceptional high value (higher than 96) for silver. This may mean that the monthly returns distribution is non-normal for this kind of assets.

5. Results

This study uses GRG Nonlinear engine for linear solver problems. **Table 5** reports the returns from each portfolio (Model 1 – Model 6) and **Table 6**, the constitution of assets for each portfolio.

Rate means the yearly return of the portfolio, and as can be seen the best result of 15,06% belongs to portfolio 2 which is expected because we are maximizing this metric (rate), although in a less consistent way since sharp ratio presents the lowest value compared to other portfolios. This means that portfolio 2 is the most volatile, i.e., in terms of sharp ratio almost equals to the benchmark (S&P 500). Still it has fewer negative years when comparing to the benchmark (3 versus 7). Although, it loses power in the good semi-decade (2011–2018), showing a return of 8,55% (annually), when in the bad decade (2000–2010) the average return was 20,03%. Portfolio 1 presents the highest sharp ratio with no negative years; the worst year presented a positive return of 2,18%. The average yearly return is 4,49% in the overall and its maximum presented a value of 6,52% (much lower comparing to 29,60% - the benchmark). It means that this portfolio is adequate to a very conservative investor. The rate is only a little higher than a deposit rate, which is expected according to its constitution (see **Table 6**) because 65% is constituted by treasury bonds, then only 16% equity, 7% commodities and 12% others (EUR/USD and VIX). In Portfolio 3 we try to create a portfolio that, in general, the rate of a bad decade is almost equal to a good decade. It is expected a yearly rate of 14,15% overall and equal for both decades. Sharp ratio still present positive values (superior to 1) and the investor should not present any negative years with Portfolio 3 (there was this condition as well in this model). To accomplish that, portfolio constitution is curious: 42% equity, 32% VIX, 26% commodities and no bonds (see **Table 6**). In Portfolio 4 the overall rate is maximized but with the constrain of the overall sharp ratio equal or superior to 1 and the solver obtained the result successfully. The overall rate is 14,83% by year which is an excellent result, but it loses "power" in the good decade (18,28% 2000–2010 vs. 10,25% 2011–2018). The worse year was in 2013 (−10,14%)

Panel A: Decade 2000–2010							
	SP500 Benchmark	**Model 1 Max SR**	**Model 2 Max return**	**Model 3 Equal return**	**Model 4 Max return/SR**	**Model 5 Model 4 (ver2)**	**Model 6 Max min**
2000	−5,32%	4,96%	29,62%	9,18%	19,89%	20,50%	8,75%
2001	−13,04%	4,02%	−13,37%	0,50%	−4,00%	−8,10%	11,23%
2002	−23,37%	4,79%	27,08%	6,96%	20,85%	12,26%	12,82%
2003	26,38%	5,31%	10,43%	12,23%	11,57%	19,94%	13,20%
2004	8,99%	4,54%	4,65%	0,50%	6,69%	1,79%	2,59%
2005	3,00%	4,21%	73,67%	37,70%	54,60%	59,50%	27,36%
2006	13,62%	3,96%	17,76%	16,80%	21,49%	16,33%	15,64%
2007	3,53%	6,08%	39,86%	32,98%	31,60%	32,30%	26,42%
2008	−38,49%	3,61%	13,02%	10,51%	8,95%	2,49%	7,33%
2009	23,45%	5,20%	9,57%	10,67%	12,29%	17,95%	18,50%
2010	12,78%	6,17%	27,57%	23,86%	26,61%	26,33%	17,49%
Rate	−0,93%	4,80%	20,03%	14,15%	18,28%	17,12%	14,44%
ER	0	5,74%	20,97%	15,09%	19,22%	18,06%	15,37%
SR	−0,05	5,97	0,94	1,22	1,24	1,00	2,00
Panel B: Semi-decade 2011–2018							
2011	−2,22%	4,76%	11,76%	14,59%	12,16%	9,32%	2,58%
2012	13,41%	4,87%	7,65%	8,29%	8,33%	8,03%	7,78%
2013	29,60%	3,64%	−13,94%	0,50%	−10,14%	−5,21%	2,58%
2014	11,39%	6,52%	12,74%	25,68%	20,72%	19,27%	24,30%
2015	−0,73%	2,70%	8,75%	17,01%	12,14%	11,00%	14,63%
2016	9,54%	2,18%	13,28%	7,31%	9,07%	16,05%	3,34%
2017	19,42%	3,09%	−5,01%	2,24%	−2,37%	1,93%	2,98%
2018	−6,24%	4,81%	41,10%	43,30%	38,71%	35,52%	32,36%
Rate	6,26%	4,06%	8,55%	14,15%	10,25%	11,42%	10,83%
ER	0	−2,19%	2,29%	7,90%	4,00%	5,17%	4,57%
SR	0,52	3,04	0,57	1,07	0,75	1,00	1,01
Panel C: All period 2000–2018							
Rate	3,14%	4,49%	15,06%	14,15%	14,83%	14,69%	12,90%
ER	0	1,35%	11,92%	11,02%	11,70%	11,55%	9,77%
SR	0,75	4,00	0,76	1,15	1,00	0,96	1,43
AVG	4,51%	4,50%	16,64%	14,78%	15,75%	15,64%	13,26%
MED	8,99%	4,76%	12,74%	10,67%	12,16%	16,05%	12,82%
MIN	−38,49%	2,18%	−13,94%	0,50%	−10,14%	−8,10%	2,58%
MAX	29,60%	6,52%	73,67%	43,30%	54,60%	59,50%	32,36%
(+)	12	19	16	19	16	17	19
(−)	7	0	3	0	3	2	0

Notes: SR = Sharp Ratio; ER = Excess return (comparing to the benchmark); AVG = Average annually returns; MED = Median annually returns; MIN = Minimum annually returns; MAX = Maximum annually returns; (+) count of positive years; (−) count of negative years.

Table 5.
Results from the 6 models.

	Model 1 Max SR	Model 2 Max return	Model 3 Equal return	Model 4 Max return/ SR	Model 5 Model 4 (ver2)	Model 6 Max min
SP 500						
DJ 30	6%					
NASDAQ						
STOX 600						
HANG SENG						
Emerging Mkts						
RE		18%	11%	31%	16%	19%
Consumer	9%		23%		3%	3%
Healthcare	1%					
Communications						
Energy						
Financials						
Industrials						
Semiconductors			9%	2%	16%	16%
B 1-3y	32%					
B 7-10y						1%
B 20 + y						1%
TIPS						2%
Corporate B						
BUND	33%					1%
Cocoa	3%		2%	2%		17%
Coffee						
Corn						
Sugar	1%					
GOLD						3%
Cooper						
Silver	2%	35%	19%	26%	27%	11%
Crude						
Natural Gas		16%	4%	9%	11%	3%
Commodities						
EUR/USD	9%					
VIX	3%	31%	32%	29%	28%	21%

Notes: SR = Sharp Ratio; SPX = Standard & Poor's ticker; DJI = Dow Jones Industrials ticker; STOXX = Eurostoxx 600 ticker; HSI = Hang Seng Index; EM = Emerging Markets; RE = Real Estate; B-ST = US Bonds Short Term; B-MT = US Bonds Medium Term; B-LT = US Bonds Long Term; TIP = Treasury Inflation Protected Securities; CorpB = Corporation Bonds; NGas = Natural Gas; EUR/USD = Euro vs. USA Dollars exchange; VIX = Volatility Index (S&P 500).

Table 6.
Constitution of the 6 models.

because metals came across with a big drop which is a big part of the constitution of this portfolio (38% commodities, which 26% silver). The remain constitution: 33% equity and 29% VIX. It is seen 3 negative years (2001, 2013 and 2017) which is

	S&P 500	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5	Portfolio 6
S&P 500	1						
Portfolio1	0,11	1					
Portfolio2	−0,23	0,30	1				
Portfolio3	−0,09	0,41	0,81	1			
Portfolio4	−0,18	0,40	0,97	0,89	1		
Portfolio5	0,04	0,35	0,94	0,86	0,95	1	
Portfolio6	−0,06	0,51	0,69	0,89	0,81	0,77	1

Table 7.
Correlation matrix of portfolios vs. benchmark.

perfectly acceptable when comparing to the benchmark (7 negative years in 19 years total). Portfolio 5 is very similar to Portfolio 4, in results and in constitution. Here, the difference is to assure a sharp ratio equal or superior to 1 for the first decade and for the second decade as well. There was a little improvement comparing to the last one, the model will "steal" 1% of the returns from the first decade and return it to the second decade, i.e., instead of 18,28% vs. 10,25% (Portfolio 4), we get 17,12% vs. 11,42%. Also, instead of 3 negative years, there is 2 negative years (2001 and 2013) and the worst year instead of −10,14% (Portfolio 4), −8,10%. Finally, Portfolio 6 we maximize the minimum return (yearly). We may say that this portfolio is an upgrade from the first one (Portfolio 1), because 1. there are no negative years, 2. the worse year presents a positive return of 2,58% and 3. it maximizes more returns to the investor. The overall return is 12,9% yearly (vs 4,49% - Portfolio 1) and sharp ratio is superior to 1 for both decades. What regards to its constitution: 38% equities, 35% commodities, 21% VIX and 5% treasury bonds.

As can be seen, all portfolios come across to the benchmark, portfolio 1 with less spikes, although, S&P500 is almost touching the line of the portfolio. Portfolio 2 seems to be the most volatile. **Table 7** shows the correlation matrix of portfolios and benchmark between themselves.

As can be seen, there is no correlation between S&P 500 and any portfolio, meaning that our proposed portfolios behave quite independently from the stock market. Portfolio 1, where we maximize the sharp ratio has no correlation with others 5 portfolios at all. Portfolio 2 to 5 are highly correlated between themselves and Portfolio 6 (max min) is highly correlated to Portfolios 3 to 5.

6. Conclusions

Our study shows that is possible to create robust portfolios where the risk is minimized, and the return is maximized. Theory behind is [2] which study focus on 'efficient frontier of optimal investment', while advocating a diversified portfolio to reduce risk. To perform it, six portfolio models are proposed, and its formation are made by a solver, where the selected solving method is the GRG Nonlinear engine for linear solver problems. Then we compare results with the benchmark (S&P 500), a linear regression model (available for request) and other "popular" portfolios (already known by many investors – also, only available by request) as well.

Results show that the GRG Nonlinear engine is powerful, providing excess returns to all six models. We design models for three types of investors: conservative, moderate and aggressive. For a conservative investor, portfolio 1 fits the best followed by portfolios 6 and 3. Portfolio 1 shows a strong sharp ratio equal to 4, presenting though, very low volatility but with lower returns when comparing to portfolios 6 and 3. None of this mentioned portfolios show a negative year during the period of 2000 to 2018. Portfolio 3 presents a high performance (14,15% annually). For an aggressive investor portfolio 2 is the best choice because it maximizes the overall return. It is the most volatile portfolio, but it may generate an average income superior to 15% annually. For a moderate investor, fan is wider but still we would exclude the portfolio 1 because it will not generate to much return and portfolio 2 may be a little volatile. Even that, portfolio 2 only presents three negatives years which is still better than the benchmark (3 versus 7).

We went further in our research and we figure out that GRG is robust, and its returns exceeds the other models mentioned in the first paragraph of this section (linear and "classical" portfolios).

Our contribution for this study is to provide a wider variety of portfolios that can be easily used by institutional and private investors and considering that nowadays there are plenty ETFs or funds available in the market is easy to everyone to apply one of the proposed models. Also, it is proved that it is possible to design very efficient portfolios, increasing returns and at the same time, lowering the risk. The results enforce the MPT from [2, 3].

As it happens in all models, there are, of course, some limitations as well. First, we may not guarantee that portfolios constitutions (1–6) will present the same results in the future because we are relaying in past returns and we would need, at least one more decade to understand if, for example, "good" decades present similar behavior between themselves. Another limitation found, the lack of the real VIX tracker (ETF/ETN). Available ETFs of VIX are a mix of mid-short term that do not reflect the actual index. Note that VIX plays an important role in GRG models.

Acknowledgements

The views expressed in this paper are those of the authors and do not necessarily represent the views of the institutions with which they are affiliated. The authors acknowledge research from the FCT (NECE: UID/GES/04630/2019). The FCT NOVA's author acknowledges Fundação para a Ciência e a Tecnologia (FCT - MCTES) for its financial support via the project UIDB/00667/2020 (UNIDEMI).

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